Laser interferometric characterization for a vibrating speaker system

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Abstract. A characterization of an oscillatory loudspeaker system is carried out via Michelson-type laser interferometry. A 632.8 nm He-Ne laser was used with a beam splitter to create an interference pattern on a photometer. The mirror of the target arm of the interferometer was attached to a loudspeaker cone. The path length difference between the two light beams incident on the photometer was then varied by means of the vibration of the mirror-loudspeaker combination. The precision of measurement of the setup was found to be on the order of λ/16 (~40) nm. Additionally, a frequency response graph of the speaker showed its natural frequency of oscillation to be 63.80 ± 0.04 Hz.

I. INTRODUCTION

The degree of precision to which small displacements are measured is becoming increasingly relevant in scientific research and industrial manufacturing. Calibration techniques in precision machining for both commercial and scientific instruments often require measurements on the order of 10^{-9} m [1]. Here we present a method that uses the oscillatory nature of a speaker to vary the phase difference of a reference and a target beam and use the corresponding interference pattern to characterize the speaker.

The invention of lasers in the early 1960s significantly improved the accuracy of optical interferometry by providing an extended monochromatic source of light for taking measurements. However, the Michelson interferometer dates back to 1887 when Albert A. Michelson and Edward W. Morley disproved the existence of a luminiferous aether by comparing the speed of light in perpendicular directions with a sodium flame [2]. Since then, numerous experiments (e.g. LIGO) have improved upon the precision of small displacement measurements to that of distances less than the diameter of a proton [3]. The Michelson interferometer presented here is a relatively inexpensive and straightforward setup that shows its powerful applications.

A common technique for measuring small displacements is the analysis of interference fringes that result from the superposition of monochromatic, homogenous plane waves. This paper describes a Michelson interferometer employing a He-Ne laser for measuring the displacement of a speaker cone at varying driving frequencies. The characterization of a loudspeaker is a sufficient demonstration of the application of a laser interferometer due to the inherent size of its displacement amplitudes. At a given driving frequency, the phase difference of the reference and target beams is directly related to the interference pattern at that point in time due to the superposition principle. Since we have two interfering plane waves each of wavelength 632.8 nm, their time-dependent superposition is represented by the intensity of the light incident to the photometer. This intensity is then converted to an electrical voltage and displayed on a digital storage oscilloscope. We will cover the relevant energy transfer functions and the corresponding intensities as a function of time. We will also examine why this method is optimal for sensitive measurements and how it can be applied to larger scale systems.

II. THEORY

A. The He-Ne laser Michelson Interferometer

Our Michelson interferometer setup used a He-Ne laser, a beam splitter, two mirrors, and a photometer, illustrated in Fig. 1. Original models used sodium flame as the coherent light source. The light propagates from the source to the beam splitter where approximately 50% of the incident light is transmitted and approximately 50% of it is reflected at a right angle. The transmitted light continues to propagate as the reference plane wave to the stationary mirror where it is fully reflected back towards the beam splitter. The reflected light propagates as the target plane wave to the mirror attached to the oscillating loudspeaker. The two waves then recombine at the beam splitter and interfere based on their relative phases based on the idea that the phase shift of the target plane wave varies with the instantaneous position of the speaker-mirror combination.

Consider a situation in which we setup our interferometer such that no phase shift corresponds to the equilibrium position of the speaker-mirror combination. At this position, we expect constructive interference at the photometer. Likewise, for every λ/2 change in displacement, we expect constructive interference at the photometer because the path length travelled by the target plane wave varies by some integral multiple of λ. Thus,

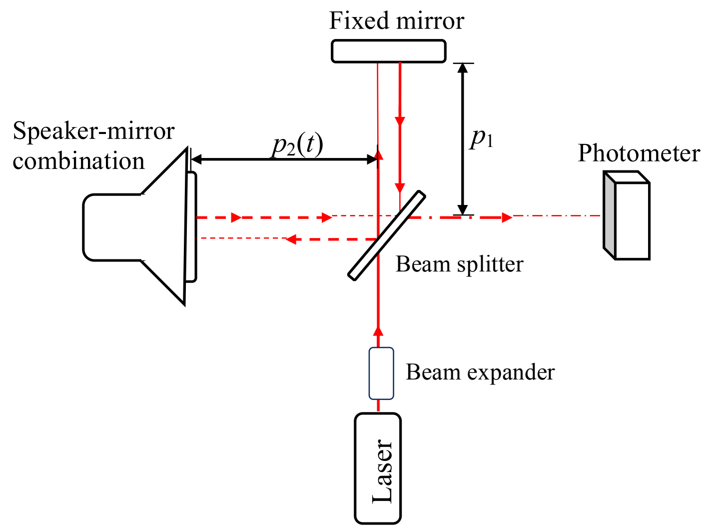

Fig. 1. Experimental setup, illustrating the Michelson interferometer and the paths of its arm lengths $p_{1,2}$. The He-Ne laser travels through a beam expander onto a beam splitter where two perpendicular waves propagate towards respective mirrors. They recombine and their interference pattern is measured by the photometer.

the phase shift is an even multiple of π and the two waves remain in phase. Similarly, when the path length traveled is some odd multiple of λ/4, the phase shift is an odd multiple of π and the two waves interfere destructively.

B. Wave mixing

Consider two coherent waves with the same amplitude A and same frequency ω that have paths x_1 and x_2 emitted from different sources that meet together at a point [4]. The net wave at that point can then be given as the real part of the equation

$$\psi = Ae^{i(kx_1-\omega t)} + Ae^{i(kx_2-\omega t)}$$
$$= Ae^{-i\omega t}e^{ik(\frac{x_1+x+2}{2})}[e^{ik(\frac{x_1-x_2}{2})} + e^{-ik(\frac{x_1-x_2}{2})}]$$
$$= 2A\cos[k(\frac{x_1-x_2}{2})]\, e^{i[k(\frac{x_1+x_2}{2})-\omega t]}, \quad (1)$$

where the wave number $k = \frac{2\pi}{\lambda} = \frac{w}{c}$, λ is the wavelength of the waves, and c is the phase velocity. The time-averaged intensity of the combined wave is then proportional to the square of the its amplitude. That is,

$$I \propto \cos^2\left[\pi\left(\frac{p_2(t)-p_1}{\lambda}\right)\right] = \cos^2\left[2\pi\left(\frac{p_2(t)}{\lambda} - \theta\right)\right], \quad (2)$$

where θ is a constant. Now suppose the mirror that the $p_2(t)$ beam is incident to oscillates harmonically back and forth with a frequency f and a displacement B. Then, $p_2(t) = p_2(0) + B\sin(2\pi ft)$ and the intensity of the detected light varies with time as

$$\mathrm{I} = A^2\, \cos^2\left[2\pi\left(\frac{B\sin(2\pi ft)}{\lambda} - \theta'\right)\right]. \quad (3)$$

Note that the frequency of oscillations is itself a sinusoid, meaning that there will be turnaround points in the oscillations [4].

III. EXPERIMENTAL REALIZATION

Our 10 mW He-Ne laser operated at a wavelength of 632.8 nm via DC electric discharge with Newport optics to expand the beam. The laser beam was projected onto the reference object and target object, a small mirror and a low-power loudspeaker, respectively. The photodiode was enclosed in a small box, which has a small hole for collecting the incoming light beam. The photometer was connected to a digital storage oscilloscope that read the intensity of incident light as a function of time. All components were mounted on an optical table in order to reduce the noise of mechanical vibrations in the setup.

Before applying the sinusoidal voltage to the loudspeaker, both the reference and target mirrors were adjusted with micrometers to obtain an interference pattern on the face of the photometer. The loudspeaker was then driven by a sinusoidal voltage of amplitude V_0 and frequency $f =$

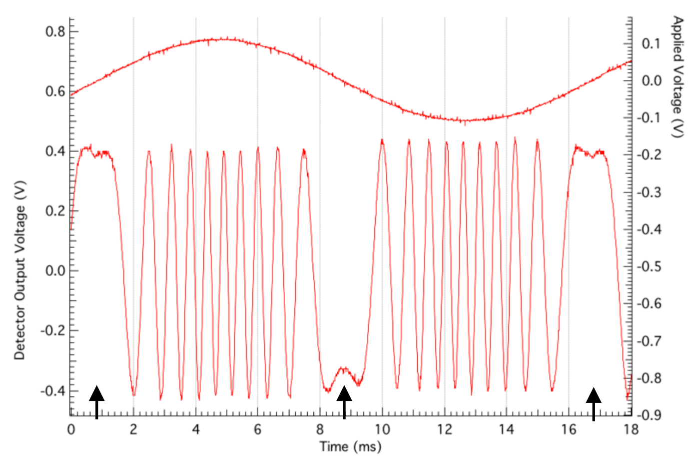

Fig. 2. Typical figure observed on the oscilloscope screen for f = 62.98 Hz and V_0 = 100 mV, as recorded by a LabView program. The curve shows the AC component of the detector output voltage. Each complete cycle of the curve corresponds to a speaker cone displacement of $\lambda/2$, with each arrow corresponding to a turnaround point.

$\omega/2\pi$ from a model 19 Wavetek signal generator. The oscilloscope was externally triggered off of the AUX output of the function generator and was set to AC coupling. The sweep time and vertical sensitivity were varied to give the maximum display for each frequency. The oscilloscope was interfaced with a LabView program to record the data.

We took data at 13 difference frequencies from 36.31 Hz to 90.37 Hz. For each of the 13 frequencies, we measured the amplitude of vibration by counting the number of peaks and valleys in a half-period of the AC signal.

IV. RESULTS AND DISCUSSION

An example of a measurement is displayed in Fig. 2 where one full period of vibration was acquired on the oscilloscope. In this example, the frequency f is 62.98 Hz and the voltage V_0 is 100 mV. The three arrows on the horizontal axis of Fig. 2 represent the locations that correspond to instantaneous zero velocity, $\frac{dp_2(t)}{dt} = 0$ at the positions $x = \pm x_0$.

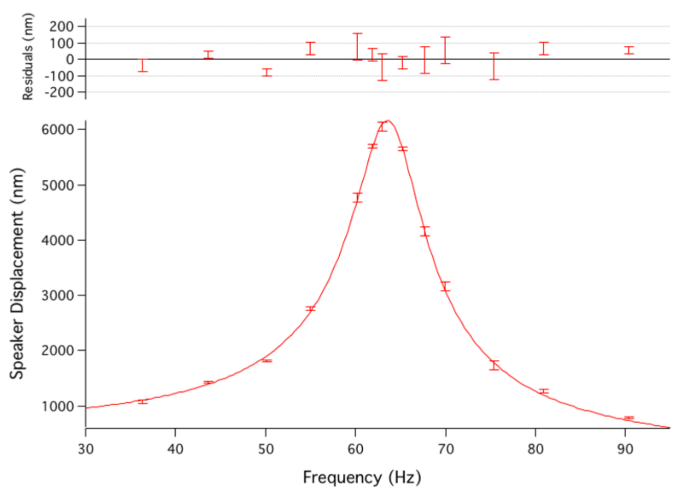

Fig. 3. Frequency response curve for the loudspeaker with corresponding residuals plot. The data were fitted to the equation for a driven, damped, harmonic oscillator.

We know from the superposition principle that any given peak-to-valley or valley-to-peak corresponds to a speaker displacement of $\lambda/4$. Thus, we can describe each peak-valley or valley-peak as a fraction of λ. Summing over all of these cycles, we can measure the displacement of the speaker at that frequency. Accordingly, we can plot a frequency response curve of the loudspeaker (Fig. 3) and thus characterize the speaker.

The equation for a driven, damped, harmonic oscillator is given by

$$|A| = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}} \quad (4)$$

where f_0 is the amplitude of the driving force, ω_0 is the resonance frequency, and β is a damping constant [5]. For any given simple harmonic oscillator, there will be a natural frequency of oscillation when the frequency is a maximum amplitude. According to our fit parameters, the natural frequency of the loudspeaker is 63.80 $\pm$ 0.04 Hz.

The uncertainty in each measurement arose out of the noise and the reproducibility of each measurement. That is, for a given peak or valley, how similar is it to the corresponding peak or valley in the previous period of oscillation? It was determined that the uncertainty in the frequency as read by the oscilloscope was too small to consider in comparison to the uncertainty in displacement.

A weighted least-squares regression found our chi-squared value to be 36.20. Our data has 10 degrees of freedom (13 data points minus 3 free parameters) so we can calculate a reduced chi-squared value of 3.62. Based on this reduced chi-squared value, we can arrive at a couple different conclusions. Either the uncertainties in each measurement were underestimated or the speaker system does not act like a perfect driven, damped, harmonic oscillator. Because of the relatively high-resolution of the data and generosity given on the uncertainties, we concluded that limitations in our equipment caused the speaker system to not act like a perfect driven, damped, harmonic oscillator. For example, the optics table which all components were mounted on was extremely sensitive to any motion which would cause our data to be slightly skewed. Additionally, the low pass filter system in the photometer may have caused some measurements to be not as exact. It is important to note that similar reduced chi-squared values were obtained by other students completing this lab, which further defends the theory of the system being an imperfect driven, damped, harmonic oscillator.

Based on our data, we can conclude that our Michelson interferometer setup can make precise measurements on the order of $\lambda/16$ (~40) nm. This conclusion is based on the fact that our oscilloscope could accurately and consistently produce a signal corresponding to this displacement, with little uncertainty to take into account.

V. CONCLUSIONS

We have shown that a speaker system can be characterized by a Michelson-type interferometer by measuring its resonance frequency. Additionally, we have shown that our Michelson interferometer setup can make precise measurements on the order of $\lambda/16$ (~40) nm. The technique of Michelson-type laser interferometry has accordingly been experimentally verified.

Further research may involve further characterizing the speaker by measuring other properties of it such as how the frequency response of a woofer relates to that of a mid-range driver relates to that of a tweeter.

VI. REFERENCES

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